# Written strategies for addition, subtraction, multiplication and division in line with the National Curriculum.
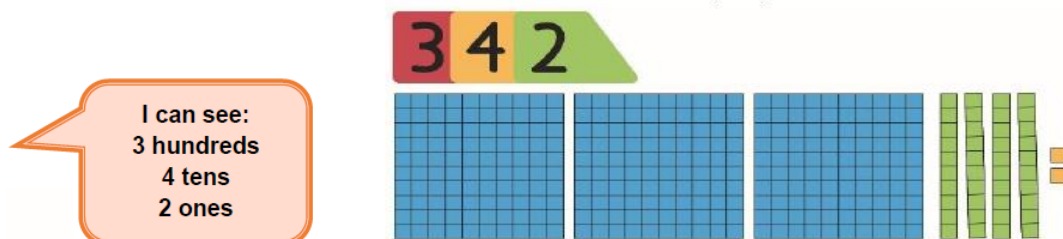
## Addition

| YEAR 3 |
| --- |
| Children are expected to be adding numbers with up to **3 digits** by the **end** of Year 3. |

**STEP 1: Use of dienes and arrow cards to understand place value and partioning.**

3 4 2

I can see:
3 hundreds
4 tens
2 ones

**STEP 2: Formal written method with no regrouping (exchange).**

```
  3 6            1 4 2
+ 4 2          + 3 3 4
  7 8            4 7 6
```

Teaching Point: Children need to know that ones line up under ones, tens under tens etc.

**STEP 3: Formal written method with regrouping of ones.**

```
+  4  3  6
         8
   4  4  4
      1
```

**STEP 4: Formal written method with regrouping tens and ones.**

```
  2 7 6          1 3 8
+   8 0        + 2 9 1
  3 5 6          4 2 9
    1              1
```

# YEAR 4
Children are expected to be adding numbers with up to **4 digits** by the **end** of Year 4.

**STEP 1:Formal addition method with no regrouping (thousands, hundreds, tens and ones).**

```
  3 5 6 9          6 0 5
+ 3 4 2 0        + 4 1 4 3
  6 9 8 9          4 7 4 8
```

**Teaching Point:**
Start by adding the ones column first.
Children need to know that ones line up under ones, tens under tens hundreds under hundreds etc.

**STEP 2 : Formal addition method with regrouping in hundreds, tens and ones.**

```
    7 9 7
  + 4 3 5
   12 3 2
    1 1
```

**STEP 3 : Formal addition method with regrouping in hundreds, tens and ones causing a further thousand.**

```
    3  7  9  7
 +  3  4  3  5
    7  2  3  2
    1     1     1
```

**YEAR 5**

Children are expected to be adding numbers with **more than 4 digits and to begin to add decimals** by the **end** of Year 5.

## Formal column method with regrouping.

```
    6 5 8 4
  + 5 8 4 8
  1 2 4 3 2
      1 1 1
```

```
    3 4 7 8 7
  +   6 7 2 3
    4 1 5 1 0
      1 1 1 1
```

**Extend to decimals.**
Use the same method to add two or more decimal fractions with up to 3 digits and the same number of decimal places.
*A Decimal Fraction is a fraction where the denominator (the bottom number) is a number such as 10, 100, 1000, etc*

**For example:**

2.3  **would look like this:**     2 and  $\dfrac{3}{10}$

**Teaching Point:** know that decimal points should line up under each other particularly when adding mixed amounts.

## Formal column method with regrouping.

```
    6 4 3 2
    4 6 8 1
      7 8 6
 +       4 2
  1 1 9 4 1
   1 1 2 1
```

**Extend to decimals.**
Use the same method to add two or more decimal fractions with up to 4 digits and either one or two decimal places.

**Teaching Point:** know that decimal points should line up under each other particularly when adding mixed amounts.

**401.2 + 26.85 +0.71 =**

```
     4 0 1 . 2
      2 6 . 8 5
 +      0 . 7 1
     4 2 8 . 7 6
```
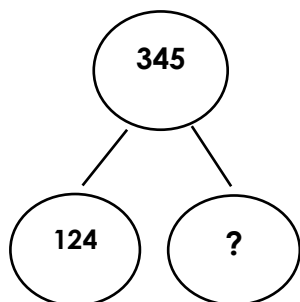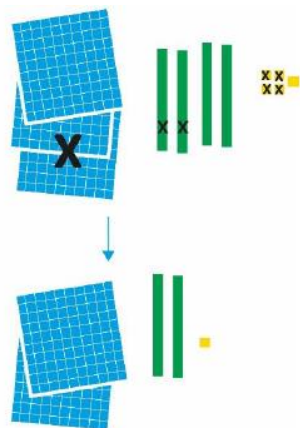
# Subtraction

## YEAR 3
Children are expected to be subtracting numbers with up to **3 digits** by the **end** of Year 3.

### STEP 1: Formal written method with no regrouping (exchange).
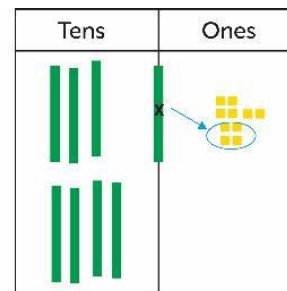
$$345 - 124 = 221$$

345
124    ?

*In subtraction, we start from the whole and take away the part we know to find the missing part.*

Dienes are used to help children understand the written method.

### STEP 2: Regrouping (exchange) tens into ones only.

$$80 - 24 = 56$$

| Tens | Ones |
|------|------|

*We need to regroup one of the tens into ten ones.*

*Now we can take 4 ones away. There are 6 ones left.*

7 1
8 0
-2 4
5 6

### STEP 3: Regrouping (exchange) hundreds into tens only.

$$825 - 241 = 584$$

7 1
8 2 5
- 2 4 1
5 8 4
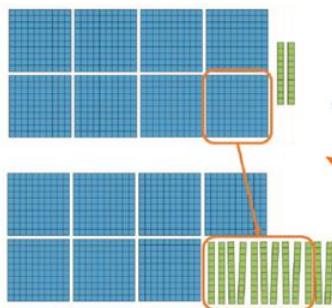
*8 hundreds and 2 tens can be regrouped into 7 hundreds and 12 tens. I take the 4 tens from 12 tens. I have 8 tens left.*

## YEAR 4

Children are expected to be subtracting numbers with up to **4 digits** by the **end of Year 4.**

**Formal written method with regrouping of thousands(decomposition).**

$$
\begin{array}{cccc}
^{2} & ^{1} & & \\
\cancel{3} & 3 & 4 & 7 \\
- \;\; 1 & 9 & 2 & 5 \\
\hline
1 & 4 & 2 & 2 \\
\hline
\end{array}
$$

**Teaching Point:** children must always subtract from the ones column first.

| YEAR 5 Children are expected to be subtracting numbers with **more than 4 digits.** | YEAR 6 Children are expected to extend the written method of subtraction with any number of digits by the **end** of Year 6. |
|---|---|
| **Column subtraction with exchanging.** | **Column subtraction with exchanging.** |

<table>
<tr><td>

**Column subtraction with exchanging.**

```
          1 1
        2 4 5 9 1
        3 5 6 0 7
      -   7 6 9 8
        2 7 9 0 9
```

**Extend to:**

- Begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places.
  For example: 24.9-13.7=

```
        2 4. 9
      - 1 3. 7
        1 1. 2
```

**Teaching point:** know that decimal points should line up under each other**.**

</td><td>

**Column subtraction with exchanging.**

```
          1 1
        2 4 5 9 1
        3 5 6 0 7
      -   7 6 9 8
        2 7 9 0 9
```

**Extend to:**

- Subtracting two or more decimal fractions with up to three digits and either one or two decimal places.

  For example: 56.91 - 24.1 - 11.71=

```
        56.91
        24.1
      - 11.71
        21.10
```

**Teaching Point:** know that decimal points should line up under each other, particularly when subtracting mixed amounts.
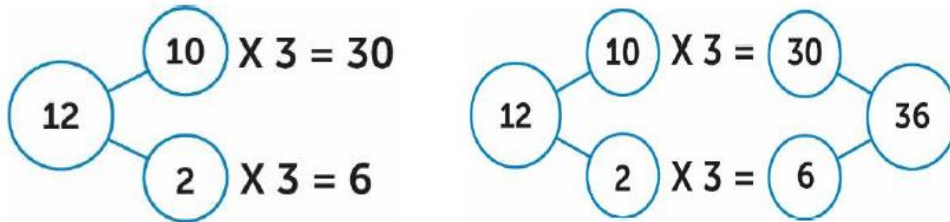
</td></tr>
</table>

# Multiplication

## YEAR 3
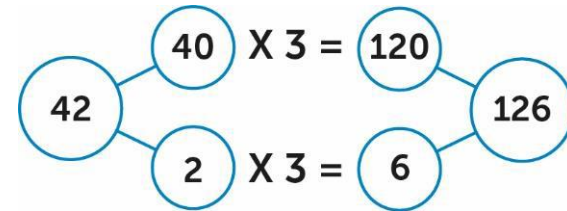Children are expected to be **multiplying two-digit** numbers by a **one-digit** number by the **end** of Year 3.

| STEP 1: Multiplying two-digit numbers by ones using distributive law (no regrouping). | STEP 2: Multiplying two-digit numbers by ones using distributive law (with regrouping). |
|---|---|
| $12 \times 3 = 36$ <br><br> 12 → 10 X 3 = 30 <br> 12 → 2 X 3 = 6 <br><br> 12 → 10 X 3 = 30, 2 X 3 = 6 → 36 | $42 \times 3 = 126$ <br><br> 42 → 40 X 3 = 120, 2 X 3 = 6 → 126 <br><br> **Teaching Points:** to regroup the number and then multiply each group by the number. <br> Add the products for the total. |
| STEP 3: Introducing short multiplication with no regrouping. | STEP 4: Short multiplication with regrouping. |
| ``` 4 2 X   3 ----- 1 2 6 ``` <br><br> **Teaching Point:** to line up the digits correctly. | <table><tr><td>tens</td><td>ones</td></tr></table> <br> 24 → 10 X 3 = 60, 4 X 3 = 12 → 72 <br><br> ``` 2 4 X   3 ----- 7 2 ``` <br><br> **Dienes and regrouping are used to help children understand the written method.** <br> **Teaching Point:** to line up the digits correctly. |

**YEAR 4**
Children are expected to be **multiplying two or three-digit** numbers by a **one-digit** number by the **end** of Year 4.

**STEP 1: Multiplying two and three-digit numbers by one-digit numbers using distributive law (with regrouping).**

52 x 3 = 156



**Teaching Points:** to regroup the number and then multiply each group by the number.
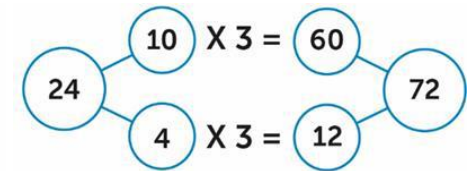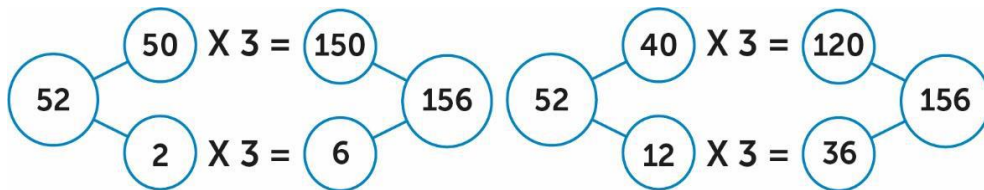Add the products for the total.

**STEP 2: Short multiplication with regrouping in one column.**

```
  1 2 4
X     3
  3 7 2
      1
```

**Teaching Point:** to line up the digits correctly.

**STEP 3: Short multiplication with regrouping in one or more columns.**

```
  1 4 6
X     5
  7 3 0
  2 3
```

# YEAR 5

Children are expected to be **multiplying up to four-digit** numbers by a **one or two digit** number by the **end** of Year 5.

---

**STEP 1: Short multiplication of a 3 or 4 digit number by a single digit.**

```
  2  1  2          1  4  3          1  2  7
x        4       x        3       x        5
  8  4  8        ¹4  2  9         ⁶ ³6  3  5
```

No regroup (exchange)     One regroup (exchanges)     Multiple regroups

**STEP 2: Long multiplication of a 3 or 4 digit number by a 2 digit number.**

```
      1  3  2
X        4  6
      1  2    (6 x 2)
      1  8  0  (6 x 30)
      6  0  0  (6 x 100)
         8  0  (40 x 2)
   1  2  0  0  (40 x 30)
   4  0  0  0  (40 x 100)
   6  0  7  2
      1  1
```

**Teaching points:**

- Children must partition the numbers correctly.
- Always write the calculations in the brackets.
- Always start by multiplying the from the ones column.

---

**STEP 3: Long multiplication of a 3 or 4 digit number by a 2 digit number.**

**STEP 1**

```
      4  3  4
x     1  2
      8  6  8
```

**STEP 2**

```
      4  3  4
x     1  2
      8  6  8
   4  3  4  0
```

**STEP 3**

```
      4  3  4
x     1  2
      8  6  8
   4  3  4  0
   5  2  0  8
      1     1
```

**Teaching points:**

- using 2 colours will prevent the children from making mistakes
- always start multiplying from the ones column.

Children are expected to **multiply multi-digit numbers up to four digits** by **a two-digit whole number** by the end of Year 6.

**STEP 1:  Long multiplication of a 3 or 4 digit number by a 2 digit number.**

|   |   | 1 | 0 | 7 | 4 |
|---|---|---|---|---|---|
| x |   |   |   | 2 | 2 |
|   |   | 2 | ¹1 | 5 | 8 |
|   | 2 | ¹1 | 4 | 8 | 0 |
|   | 2 | 3 | 6 | 2 | 8 |
|   |   |   |   | 1 |   |

**Teaching points:**
- Using 2 colours will prevent the children from making mistakes.
- Always start multiplying from the ones column.

**STEP 2: Written method of multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number.**

|   |   | 3 | 4 • | 2 |
|---|---|---|---|---|
| x |   |   | 6 |   |
| 2 | ²0 | ¹5 • | 2 |   |

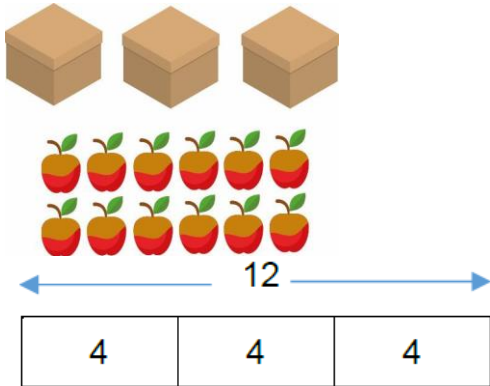**Teaching points:**
Line up the digits and the decimal point.

# Division

Children are expected to be dividing **two digit** numbers by a one-digit number with remainders by the **end** of Year 3.
**(Children need to recall their multiplication tables accurately).**
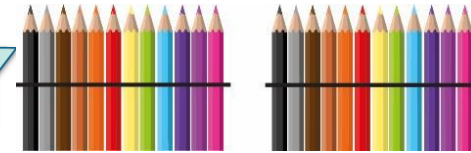
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**STEP 1: Division by sharing.**

$12 ÷ 3 = 4$

12

| 4 | 4 | 4 |
|---|---|---|

**Pictorial representation**          **Bar Model**

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**STEP 2: Division by grouping.**

Mrs Khindey has 24 colouring pencils. She bundles them into groups of 12. How many bundles will she have?

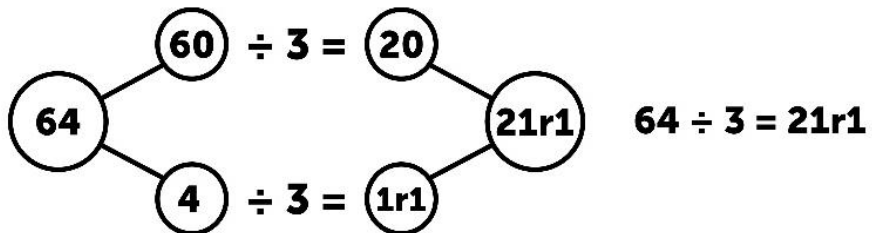We know the size of the group but not the number of groups. We are dividing by grouping.

$24 ÷ 12 = 2$
12 is the size of each group and 2 is the number of groups.

We can use the fact
$2 × 12 = 24$

There are 2 groups of 12 in 24.

---

**STEP 3: Sharing 2 and 3 digit numbers by ones with no regrouping.**

$60 ÷ 3 = 20$

$64$

$4 ÷ 3 = 1r1$

$21r1$

$64 ÷ 3 = 21r1$

---

**STEP 4: Introducing the long division method (sharing ones).**

$13 ÷ 4 = 3$ remainder 1

```
        3  r 1
   4 | 1  3
   -   1  2
   ─────────
          1
```

| STEP 5: Long division of tens and ones with no regrouping. | STEP 5: Long division of tens and ones with regrouping. |
|---|---|
|  |  |

**Teaching points:**
- Children to use base facts to help them.
- Children will be reminded about place value- for example how many groups of 2s are there in 80 or 8 tens.

## STEP 1: Long division with regrouping hundreds into tens.

426 ÷ 3= 142

quotient- the answer
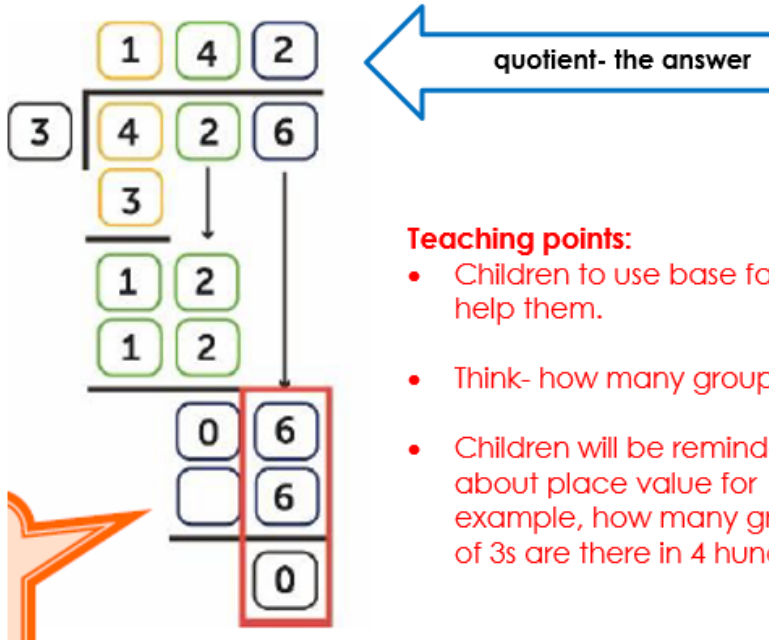
**Teaching points:**
- Children to use base facts to help them.

- Think- how many groups?

- Children will be reminded about place value for example, how many groups of 3s are there in 4 hundred?

## STEP 2: Long division with regrouping hundreds into tens and tens into ones.

880 ÷ 3= 293 r 1
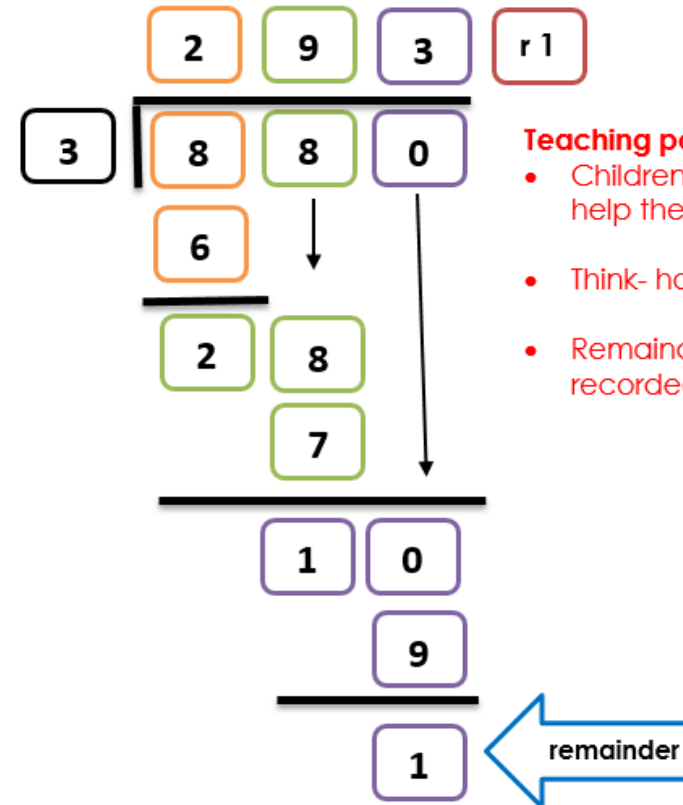
**Teaching points:**
- Children to use base facts to help them.

- Think- how many groups?

- Remainders need to be recorded as whole numbers.

remainder

## STEP 1: Short division explained.

**How many groups of 7?**

| | 7 | 6 | 7 | 7 | 6 |
|---|---|---|---|---|---|

The calculation is 6776 ÷ 7.
I want to know how many groups of 7 are in 6776.
I am going to list the multiples of 7 to help me.

| | | 0 | | | |
|---|---|---|---|---|---|
| | 7 | 6 | ⑦ | 7 | 6 |

I have no (0) groups of 7 thousands in 6 thousands.
6 thousands have to be regrouped.
I now have 67 hundreds.

| | | 0 | ⑨ | | |
|---|---|---|---|---|---|
| | 7 | 6 | ⁶7 | ⁴7 | 6 |

From my jottings, I can see that ⑨ x 7 = 63.
There are ⑨ groups of 7 hundreds in 67 hundreds.
4 hundreds have to be regrouped so now I have 47 tens.

| | | 0 | 9 | ⑥ | |
|---|---|---|---|---|---|
| | 7 | 6 | ⁶7 | ⁴7 | ⁵6 |

From my jottings, I can see that ⑥ x 7 = 42.
There are ⑥ groups of tens in 47 tens.
5 tens have to be regrouped so I have 56 ones.

| | | 0 | 9 | 6 | ⑧ |
|---|---|---|---|---|---|
| | 7 | 6 | ⁶7 | ⁴7 | ⁵6 |

From my jottings, I can see that ⑧ x 7 = 56.
There are ⑧ groups of 7 in 56 ones.

| | |
|---|---|
| | 7 |
| 1 | 4 |
| 2 | 1 |
| 2 | 8 |
| 3 | 5 |
| 4 | 2 |
| 4 | 9 |
| 5 | 6 |
| 6 | 3 |

So, there are 900 + 60 + 8 groups of 7 in 6776.
There are 968 groups of 7 in 6776.
7 x 968 = 6776      6776 ÷ 7 = 968

## STEP 2: Interpreting a remainder as a fraction.

**75 ÷ 6 = 12 remainder 3**

What are you dividing by?
What does that remainder represent?

$$1 \quad 2 \text{ r } 3$$
$$6 \overline{)7 \; 5}$$

I have divided the whole number by 6, so I want to divide my remainder by 6 too.

3 ÷ 6 can be written as the fraction $\frac{3}{6}$      $75 \div 6 = 12\frac{3}{6}$

I know that $\frac{3}{6}$ is equivalent to $\frac{1}{2}$ so $75 \div 6 = 12\frac{1}{2}$

## STEP 3: Interpreting remainder as a decimal.

| | 1 | 2 · 5 |
|---|---|---|
| 6 | 7 | ¹5 · ³0 |

The 3 ones can be regrouped to give 30 tenths.
There are 5 groups of 6 tenths.

We said that the remainder of 3 could be expressed as **3/6** or **1/2**.

I know **1/2** = 0.5 so 75 ÷ 6 = 12.5.

# YEAR 6
Children are expected to be dividing numbers **up to four digits** by a **two- digit** whole number including **remainders by** the **end** of Year 6.
Interpret remainders as whole number remainders, fractions, decimals or by rounding.

## STEP 1: Long Division Explained  6776 ÷ 7 =

**How many groups of 7?**

| 7 |
|---|
| 1 4 |
| 2 1 |
| 2 8 |
| 3 5 |
| 4 2 |
| 4 9 |
| 5 6 |
| 6 3 |

```
7 | 6 7 7 6
```

```
    0 9
7 | 6 7 7 6
    6 3
        4
```

The calculation is 6776 ÷ 7.
I want to know how many 7s in 6776.
I am going to list the multiples of 7 to help

There are 0 groups of 7 thousands so I regroup to 67 hundreds.
9 x 7 = 63 so there are 9 groups of hundreds.

There are 4 leftover.
4 hundreds have to be regrouped.

```
    0 9 6
7 | 6 7 7 6
    6 3
        4 7
        4 2
            5
```

Instead of carrying the 4 at the top, I am going to pull down the 7 to show the 47 tens which I have now.

From my jottings, I can see that 6 x 7 = 42.
There are 6 groups of tens.

There are 5 leftover.
5 tens have to be regrouped.

```
    0 9 6 8
7 | 6 7 7 6
    6 3
        4 7
        4 2
            5 6
            5 6
                0
```

Instead of carrying the 5 at the top, I am going to pull down the 6 to show the 56 ones which I have now.

From my jottings, I can see that 8 x 7 = 56.
There are 8 groups of ones.

There are none leftover.

**STEP 2: Expressing remainders as fractions.**

$$
\begin{array}{r}
120 \\
12\,\overline{)1446} \\
-12\phantom{46} \\
\hline
24\phantom{6} \\
-24\phantom{6} \\
\hline
06 \\
\end{array}
$$

I would use long division to show 1,446 ÷ 12 = 120 remainder 6.

I need to divide the remainder (6) by the divisor (12).

6 divided by 12 can be written as $\frac{6}{12}$ and that is equivalent to $\frac{1}{2}$.

So 1,446 ÷ 12 = $120\frac{1}{2}$

# STEP 3: Expressing remainders as decimals in long division.

How many groups of 12?

$$12\,\overline{)1446}$$

| 1 | 2 |
|---|---|
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |
| 6 | 0 |
| 7 | 2 |
| 8 | 4 |
| 9 | 6 |
| 1 | 0 | 8 |

$$12\,\overline{)1446}$$
-12

(2)

$$12\,\overline{)1446}$$
-12
24
-  24

The calculation is 1,446 ÷ 12.
I want to know how many 12s in 1,446.
I am going to list the multiples of 12 to help me.

There are 0 groups of 12 thousands so I regroup to 14 hundreds.
I know 1 x 12 = 12 so there is 1 group of 12 hundreds and I can work out that there are 2 hundreds remaining to regroup.

Instead of carrying at the top, I am going to pull down the 4 to show the 24 tens which I have now.
I know 2 x 12 = 24 so there are 2 groups of 12 tens and none to regroup.

$$12\,\overline{)1446}$$  → 120
-12
24
-  24
06

I still have 6 ones left which I drop down.
I can make 0 groups of 12 from 6 ones.
What next?

$$12\,\overline{)1446.0}$$  → 120.5
-12
24
-  24
06.0
6.0
0.0

I can regroup the 6 ones to make 60 tenths.
There are 5 groups of 12 in 60 tenths.
There is 0 left.
1,446 ÷ 12 = 120.5