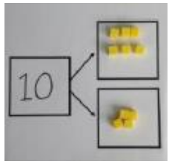
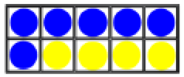
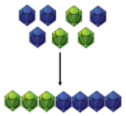
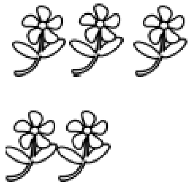
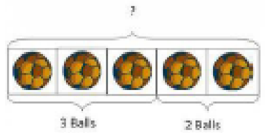
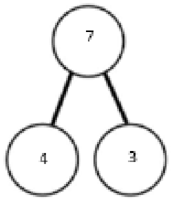
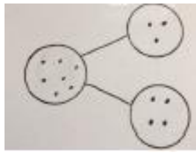
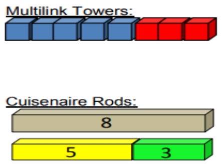
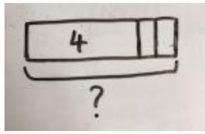
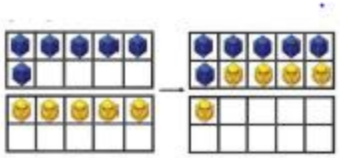
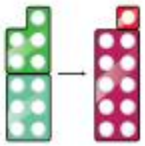
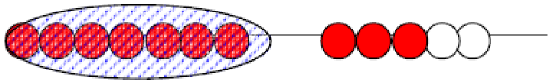
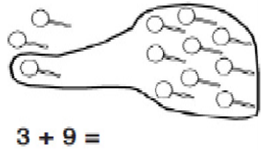
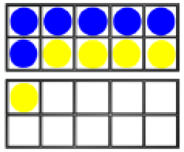
Written strategies for addition, subtraction, multiplication and division in line with the National Curriculum.

Addition

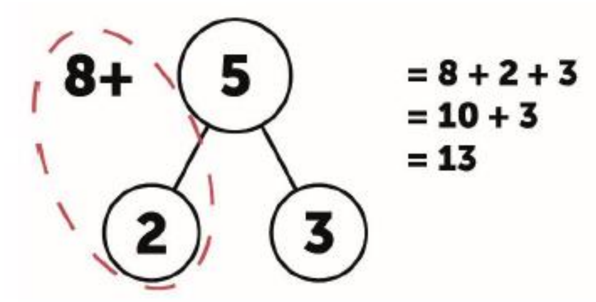
Addition and Subtraction are connected. Addition names the whole in terms of parts, while subtraction names a missing part of the whole.

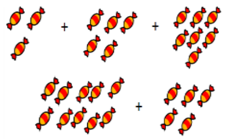
Part	**Part**
Whole	

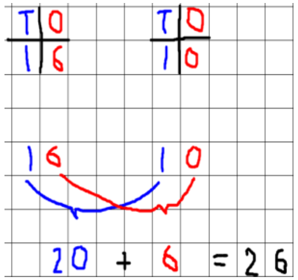
Year 1			
Objective and strategies	**Concrete**	**Pictorial**	**Abstract**
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or a bar (aggregation). Start counting at 1. Count one set and then the other. Then count them altogether. (Use other resources – eggs, shells, cars etc) Using 10s frames to add two amounts using two different colours for the 2 numbers.  	 Use pictures to add two numbers together. Use the bar model to add two numbers together.  Children draw crosses, dots or numbers in a part-whole model and add together.  	Children start to show recognisable abstract number sentences alongside the pictorial and concrete creations. $4 + 3 = 7$ $10 = 6 + 4$ (Equal sign does not have to come at the end).

Starting at the bigger number and counting on	**This stage is essential.** Children start to calculate  rather than just count. Where one quantity is increased by some amount (augmentation). Count on from the total of the first set (3 in your head) and count on 2. Always start with the larger number. Use bead strings or Cuisenaire Rods. Number tracks teach children the order of numbers. Number line - points are marked instead (allowing fractions of numbers). Could write number sentence along with creation.	Use a bar model that encourages the children to count on rather than  count the whole.	Children start to show recognisable abstract number sentences. The sum is 4 + 2 = Or, = 4 + 2 Not, 2 + 4
Regrouping to make 10	Use ten frames and counters/cubes or use Numicon. 6+5 = 11   Use bead strings to show 7 + 5 can be partitioned into 7 + 3 + 2 (children use number bonds to 10). 	Children then draw the ten frame Or, use their own pictures to show regrouping  	The sum is 9 + 5 = Children develop an understanding of equality: $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

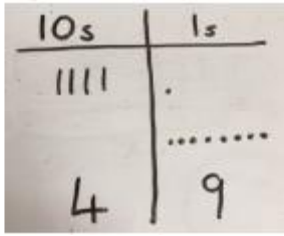
<table>
<tr>
<td>Adding 10 and then compensating</td>
<td>Use a bead string to work out sums
e.g. 7 + 9

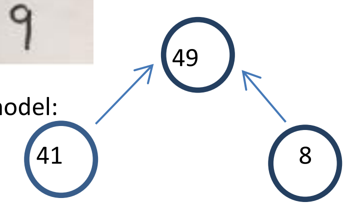
Children find 7, then add 10 and then adjust by removing 1.</td>
<td>Children would use the idea of think 10 and draw the part part whole and think 10. For example, The 5 has been regrouped into 2 and 3. The 2 has been added to the 8 to make 10 and then 3 more is added.

</td>
<td>Children would write the sum out.
E.g. 8+5=13</td>
</tr>
<tr>
<td>Adding in any order</td>
<td>Explore the commutative law, where you can add in any order, using cubes.</td>
<td>Draw the cubes to show that the total has not changed.</td>
<td>6 + 3 + 4 is the same as 6 + 4 + 3 however the second number sentence is easier & quicker.</td>
</tr>
<tr>
<td colspan="4">Children should be here by the end of year 1</td>
</tr>
<tr>
<td colspan="4">Year 2</td>
</tr>
<tr>
<td>Adding three single digits</td>
<td>Use bead strings to work out sums e.g.
4 + 7 + 6 = 17. Put 4 and 6 together to make 10.
Add on 7.

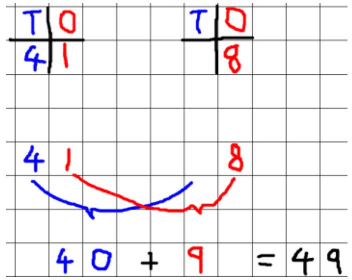
Build a tower of bricks and then ask the child to split them in 3 ways – add the numbers together.</td>
<td>Add together three groups of objects. Draw a picture to recombine the groups to make 10.

Bar models represent the 3 numbers.
<table><tr><td colspan="3">?</td></tr><tr><td>10</td><td>20</td><td>30</td></tr></table></td>
<td>Combine the two numbers to make 10 and then add on the remainder.
$4 + 7 + 6 = \boxed{10} + \boxed{7}$
10
$= \boxed{17}$
Children should be shown missing number sentences.</td>
</tr>
</table>

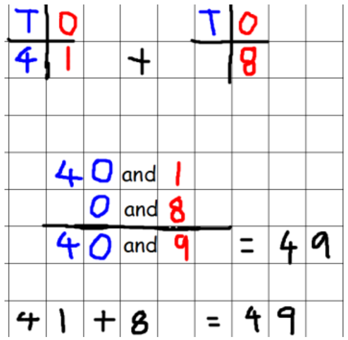
<table>
<tr>
<td>Adding 10</td>
<td>Use cubes or dienes to start with a number and add on 10 more.</td>
<td>Use or draw number squares to count on 10 more by looking at the number directly below.</td>
<td>Children to use informal partitioning method:


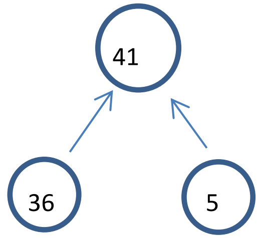
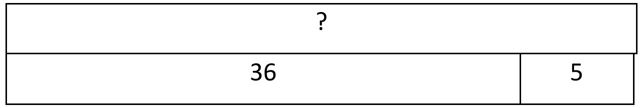
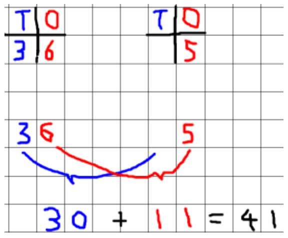
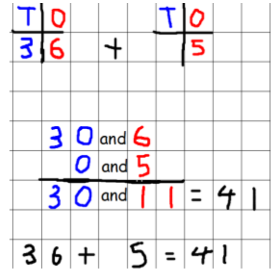
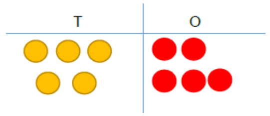
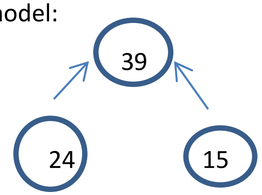
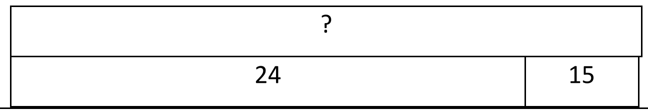
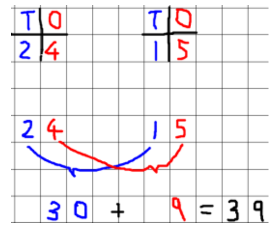
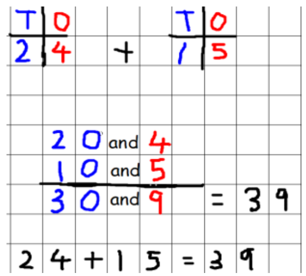
Place larger number in your head and add on 10 more by counting in tens.
Represent the number sentence in different ways: 41 = 31 +10, 31 + 10 = 41</td>
</tr>
<tr>
<td>TO + O
(No regrouping)</td>
<td>Continue to develop understanding of place value and partitioning e.g. 41 + 8

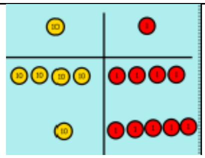
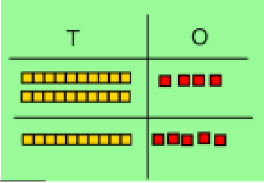
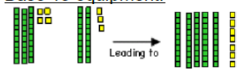
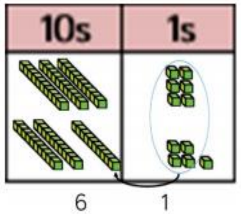
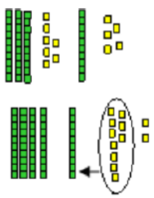
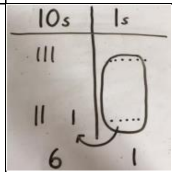
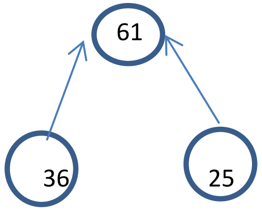
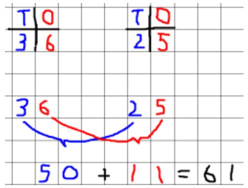
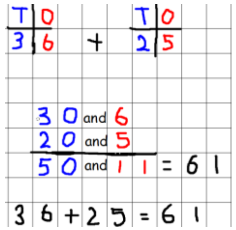
Using dienes or Cuisenaire rods to show bar models.</td>
<td>Represent base 10 with lines / dots
e.g. 41 + 8


The part whole model:


The bar model:
<table><tr><td colspan="2">?</td></tr><tr><td>8</td><td>41</td></tr></table></td>
<td>Children to use informal partitioning method:


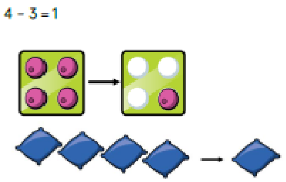
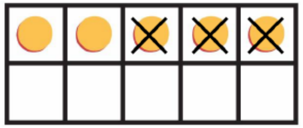
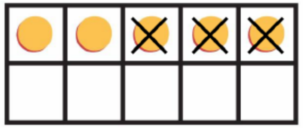
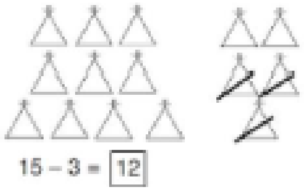
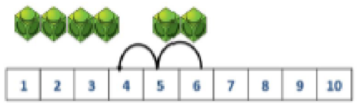
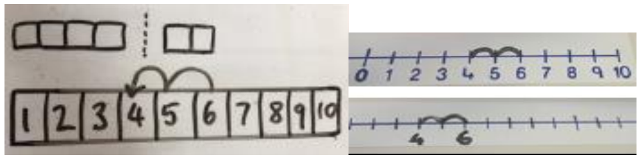
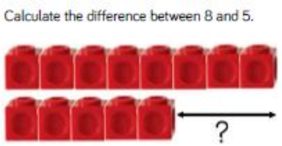
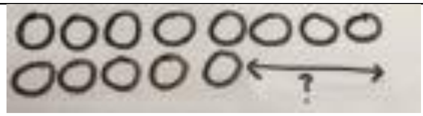
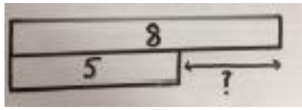
Introduction of the partitioning column method:

</td>
</tr>
</table>

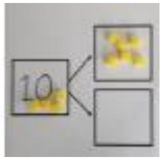
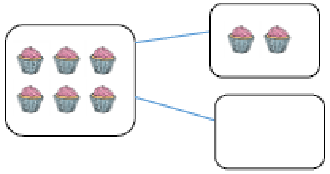
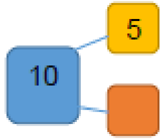
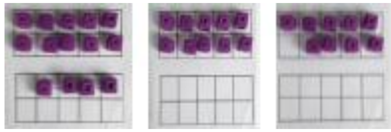
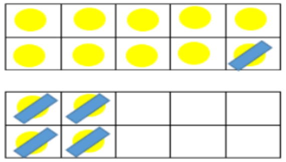
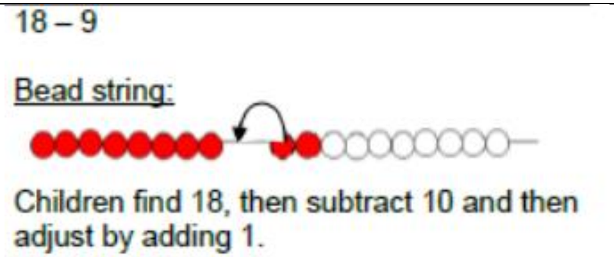
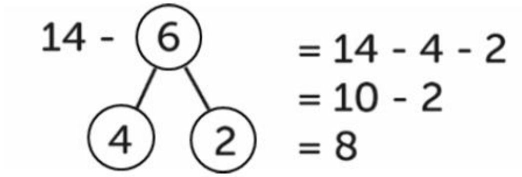
TO + O **(With regrouping in the ones)**	Continue to develop understanding of partitioning and place value 36 + 5 Using dienes or Cuisenaire rods to show bar models.	Represent base 10 with lines / dots e.g. 36 + 5 The part whole model:  The bar model: 	Children to use **informal partitioning method:**  Introduction of the **partitioning column method:** 
TO + TO **(No regrouping)**	Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. E.g. 24 + 15	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  The part whole model:  The bar model: 	Children to use informal **partitioning method:**  Introduction of **partitioning column method:** 

	  **<u>Partitioning (Aggregation model)</u>** 34 + 23 = 57 <u>Base 10 equipment:</u>  Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens. Using dienes/Cuisenaire to show bar models.		
TO + TO **(With regrouping in the ones)**	Show regrouping using dienes: 36 + 25  37 + 15 = 52  What are the missing counters and digits problems:  Use Cuisenaire rods to show bar models.	 Children to represent base 10 by drawing it in a place value chart. The part whole model:  The bar model: <table><tr><td colspan="2">?</td></tr><tr><td>36</td><td>25</td></tr></table>	Children to use informal **partitioning method:**  Introduction of **partitioning column method:** 

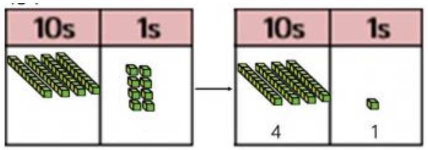
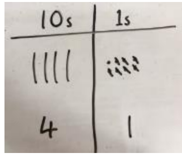
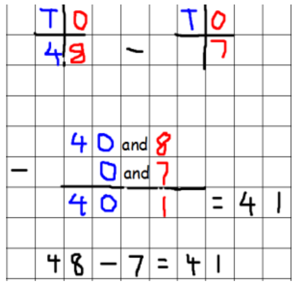
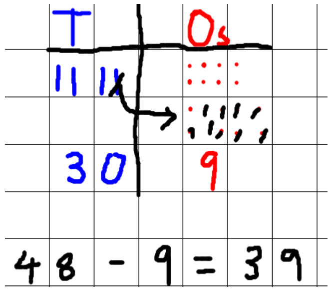
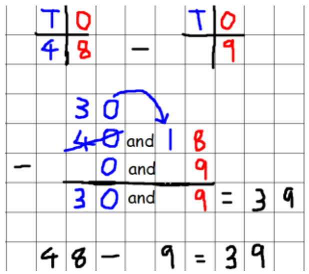
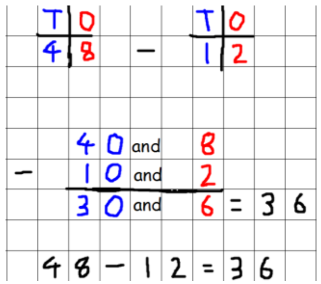
Children should be here by the end of Y2

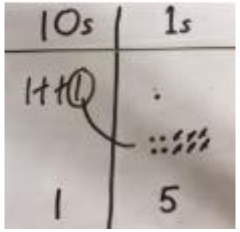
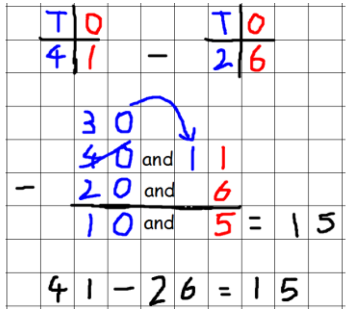
Subtraction

Objective and strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects: counters, cubes, 10s frames and counters etc to show how objects can be taken away (separation model).  5-3=2 	Cross out drawn objects to show what has been taken away   	Children start to show recognisable abstract number sentences. 4 – 3 = 1 5-3=2
Counting back	Using number lines or number tracks. Children start with 6 and count back 2  	Represent on number line (full and empty). 	Children start to show recognisable abstract number sentences. 6 – 2 = 4
Find the difference	Finding the difference using cubes, bead strings Numicon or Cuisenaire rods (comparison model).  	Draw the cubes  Use the bar model  	Find the difference between 8 and 5 8 – 5 = Explore why 9 – 6 = 8 - 5

Part whole model	Link to addition – use the part whole model to help explain the inverse. Explore using counters and bead string. 	Use a pictorial representation of objects 	Use numbers within the part whole model 
Make 10 (bridging 10) by partitioning one of the numbers	14-5 (Numicon, counters, 10 square, bead string)  Take away 4 to make 10 Then takeaway 1 so you have taken away 5. You are left with the answer of 9.	Ten frame: crossing out how many they need to rake away. 	Children start to show recognisable abstract number sentences. 13 - 7 = 6, 13 – 6 = 7 14 - 5 = 9 5 5 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5 14 - 5 = 9 4 1 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9
Subtracting 10 and then compensating		Children use the think 10 method and split the 6 into numbers that make 10. 	

Children should be here by the end of Year 1

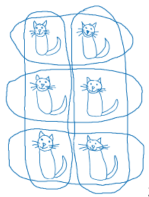
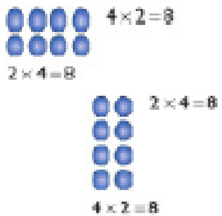
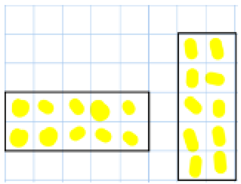
Year 2			
TO – O **(No regrouping)**	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 7 Children should be advised to use mental methods to calculate this sum initially, before proving their answer with written methods. 	Draw the base 10/place value counters and then cross out what you are subtracting.  The bar model: 48 ? | 7	Introduction of the **partitioning column method:** 
TO – O **(With regrouping in the ones)**	Create the bigger number using base 10/place value counters and then subtract the smaller number. You can't remove 9 from 8, so you need to 'steal' a ten from the next column. Regroup into 30 and 18. Children can play around with numbers that can add to 48. 48 – 9	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 	Introduction of the **partitioning column method:** 
TO – TO **(No regrouping)**	Create the bigger number using base 10/place value counters and then subtract the smaller number. 48 – 12	Draw the base 10/place value counters and then cross out what you are subtracting. The bar model: 48 ? | 12	Introduction of the **partitioning column method:** 

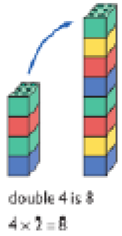
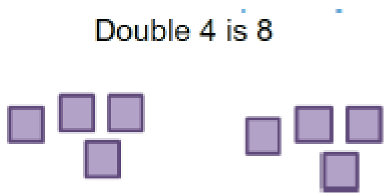
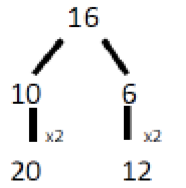
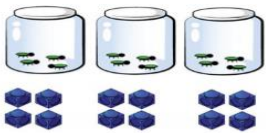
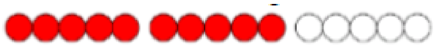
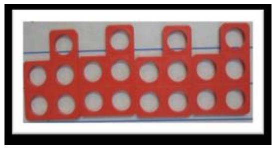
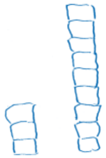
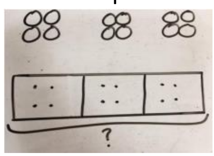
TO – TO **(With regrouping in the ones)**	Create the bigger number using base 10/place value counters and then subtract the smaller number. 41 - 26	Draw the base 10/place value counters and then cross out what you are subtracting. The regrouping must be clearly shown. 41 - 26 	Introduction of the **partitioning column method:** 
Children should be here by the end of Year 2			

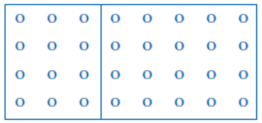
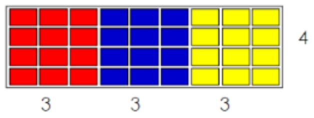
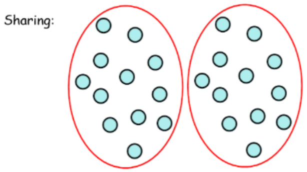
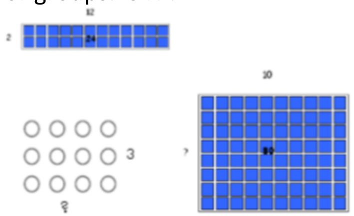
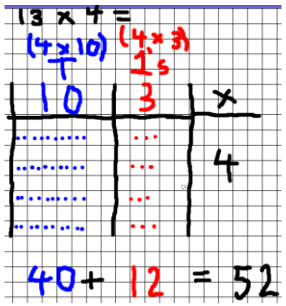
Multiplication

Multiplication and division are connected. Both express the relationship between a number of equal parts and the whole.

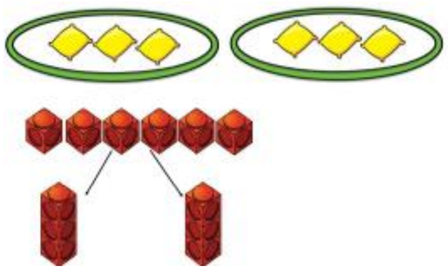
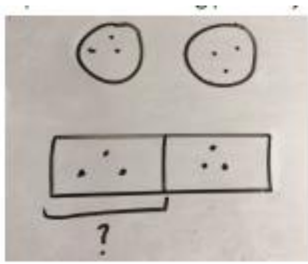
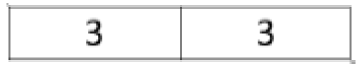
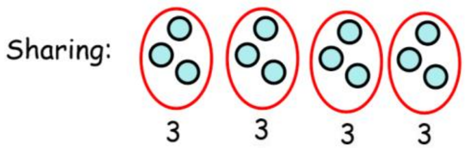
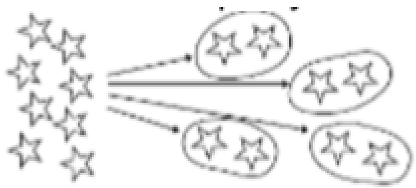
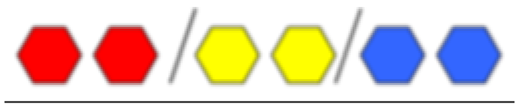
Part	Part	Part	Part
Whole			

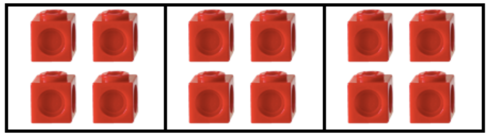
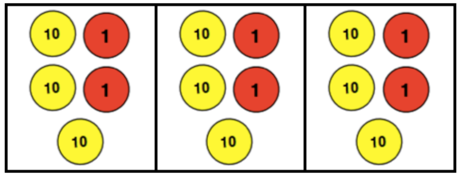
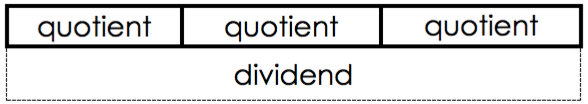
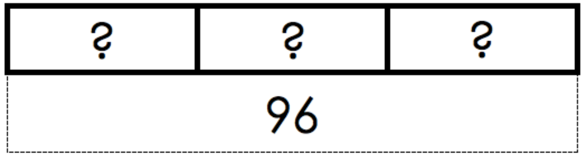
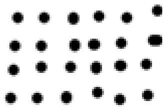
<u>Objectives and strategies</u>	**<u>Concrete</u>**	**<u>Pictorial</u>**	**<u>Abstract</u>**
Multiply by adding equal groups together	Use a set of objects. Double the set by finding the same number again. Make sure both sets are equal.	   Draw the objects  showing: 2 x 3 and 3 x 2	Children may start to show recognisable abstract number sentences. 3 x 4 = 12 4 + 4 + 4 = 12
Introduction of using arrays to count in multiples of 2, 5, 10 (commutative law)	Use a set of objects. Children can place them in groups  or start to focus them in on array shapes.  2 x 6 6 x 2	Draw the objects in arrays. Draw in different rotations to find the communtative sentences. This prepares children for the grid method and finding of factors. Also, to help find the area of rectangles.  	Children count in multiples of a number out loud. (See mental mathematics policy for more information). Write sequences with multiples of numbers. 2, 4, 6, 8 etc Children start to use an array to write a range of abstract calculations. 2 x 5 = 10, 5 x 2 = 10, 5 + 5 = 10, 2 + 2 + 2 + 2 + 2 = 10

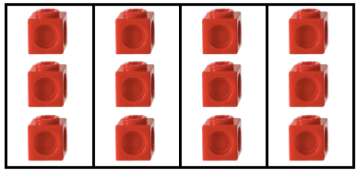
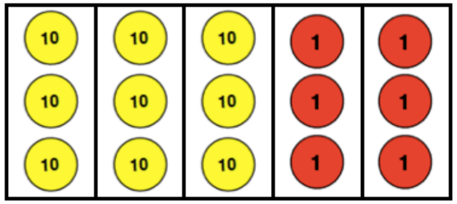
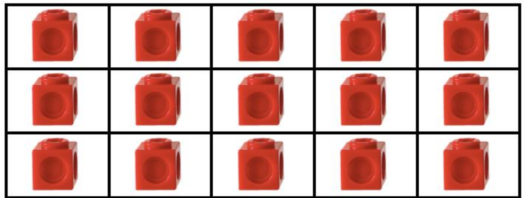
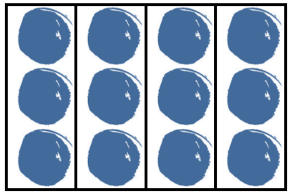
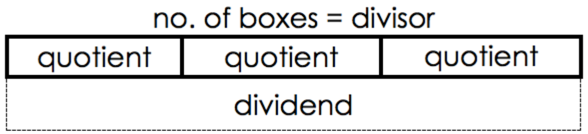
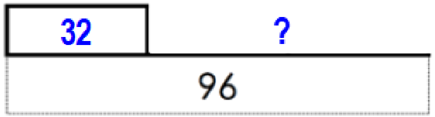
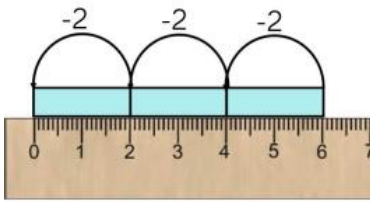
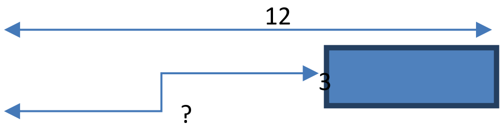
Reason about odd and even numbers and relate to doubling and halving	Create arrays of odd and even numbers with objects – what is the same or different about them? Double the number by adding the same number of objects and discuss what happens.	Draw the objects and circle/highlight the differences and similarities. Draw what happens when you double the number.	Children may start to show abstract number sentences. 3 + 3 = 6 Odd + Odd = Even
Doubling of all numbers up to 10/ **halving**	Use practical activities to show how to double a number. 	Draw pictures to show how to double a number 	 Partition a number, then double each part before recombining
Repeated grouping / repeated addition	There are 3 equal groups with 4 in each group.  Use a bead string to show repeated addition.  Children use Cuisenaire Rods to partition totals into equal trains.  Using Numicon to show 4 x 5: 	Make a necklace with red and yellow beads using three red beads for every yellow bead. Use the bricks to make a tower three times as high as this one:  Children represent the practical resources in a picture and use a bar model. 	Children start to show recognisable abstract number sentences.  2 + 2 + 2 + 2 + 2 = 10 Children are taught about the multiplication 'x' symbol. 3 x 4 = 12 is the same as 4 + 4 + 4 = 12
Children should be here by the end of Year 1			

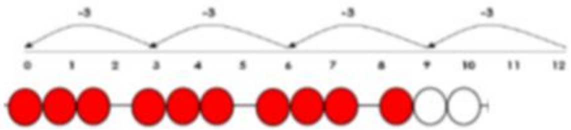
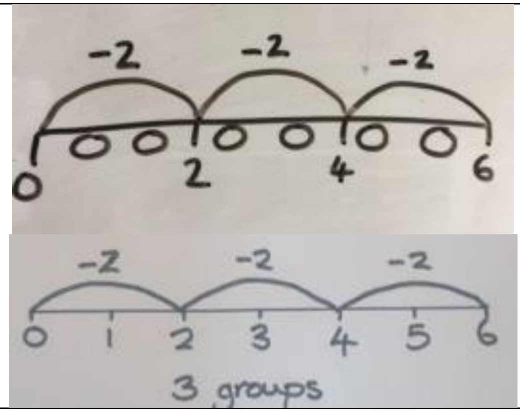
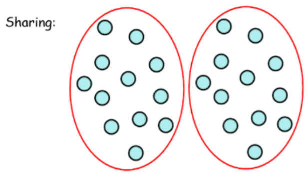
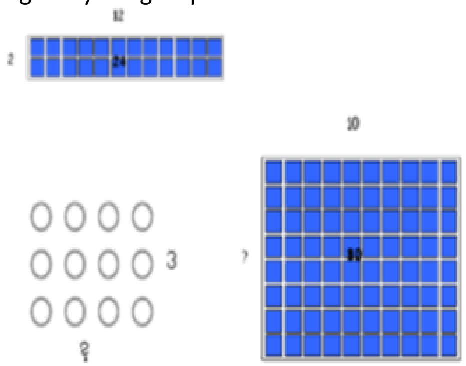
Year 2			
Consolidating use of arrays and repeated addition (distributive law)	32 pegs on a board are to be arranged into fours. How can these be shown? This shows the distributive law where 8 x 4 = 3 x 4 + 5 x 4. 	Ch to illustrate this in different ways and should be encouraged to be flexible with how they use number and can be encouraged to break the array into more manageable chunks. 9 x 4 =  	Which could also be seen as 9 x 4 = (3 x 4) + (3 x 4) + (3 x 4) = 12 + 12 + 12 = 36 Or 3 x (3 x 4) = 36
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24  	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
TO x O (No regrouping)	Use different resources to create the arrays.	Starting to organise and therefore draw arrays in columns and  show in a bar model. 52 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4	
Children should be here by the end of Year 2			

Division

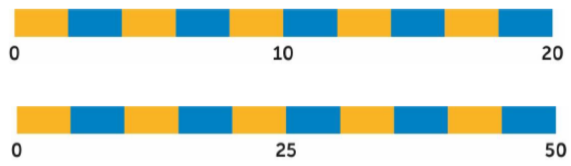
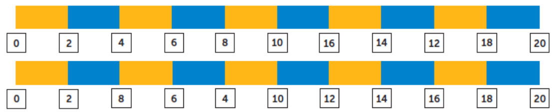
Objectives and strategies	Concrete	Pictorial	Abstract
Equal groups	Children will group different objects into equal sized piles.	Children will draw equal sized groups of objects.	
Sharing objects into equal sized groups	I have 6 cubes; can you share them equally into 2 groups? 	Represent the idea pictorially and using a bar.  	6 ÷ 2 = 3 Children should be encouraged to link these ideas to their times tables facts. Ch could draw bars with abstract numbers in them. 
Solve problems which involved sharing or grouping **Know all halves to 10 through grouping and sharing**	Sharing: Introduce practical problems which the children can physically solve. 1) Look at the number that we are dividing e.g. 12 2) Share this number out equally into section of the number that we are dividing by e.g. 4 3) Count how many there are in each section 12 sweets get shared 4 people, how many sweets does each person get? 12 ÷ 4 = 3  Grouping: 1) Look at the number that we are dividing e.g. 12	Draw a picture to show what happened. Sharing:  Grouping: 	

	2) Count or draw this many objects 3) How many groups of the number we are dividing by (e.g. take 4 objects and make one group) can you make? 4) Count how many groups you have made		
Children should be here by the end of Y1			
Year 2			
Sharing objects into groups	Share objects into groups. I have 12 cubes.  Can they be shared equally in 3 groups? After sharing between 3 groups we have found that are 4 in each group.  12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After sharing we found there were 3 tens and 2 ones in each group.	Use pictures or shapes to share quantities.  12 ÷ 3 = 4 Bar Modelling: Split the bar into the number of groups you are dividing by and work out how many would be within each group. Children do not need to use these words! no. of boxes = divisor  dividend ÷ divisor = quotient e.g. 96 ÷ 3 = 	Share 12 sweets between 3 people. 12 ÷ 3 = 4 Share £96 between 3 children. £96 ÷ 3 = £32
Grouping objects	Divide quantities into equal sized groups. I have 12 cubes.  After making groups of 3 we discovered there were 4 of them.	Represent using arrays: How many strawberries will each child have if 30 are shared between 5 children? 30 ÷ 6 = 5  30 ÷ 5 = 6	Sweets are sold in bags of 3. If I have 12 sweets how many bags would I need? 12 ÷ 3 = 4 There are 96 children sitting in rows of 3. How many rows are there?

	 12 ÷ 3 = 4 Using place value counters e.g. 96 ÷ 3 = 32  After making groups of 3, we find there were 3 groups of ten and 2 groups of one. Creating different arrays using cubes.  Division: 15 ÷ 3 = 5 15 ÷ 5 = 3 There of 5 groups of 3 in 15 There are 3 groups of 5 in 15 Multiplication: 5 x 3 = 15 3 x 5 = 15 5 groups of 3 is 15 3 groups of 5 is 15	Arrays are really important as they link to the bus stop method!  12 ÷ 3 = 4 Bar Modelling: You know how many would be within each group, but need to find out how many groups.  dividend ÷ quotient = divisor 96 ÷ ? = 32  	96 ÷ 3 = 32
Grouping using repeated subtraction	Using Cuisenaire rods above a ruler. Discuss that the number sentence (6 ÷ 2 = ?), says, "How many 2s fit into 6?" How big is each hop/rod?  3 groups of 2	Represent using a bar model and link to the Cuisenaire rods and bead strings: 12 ÷ ? = 3  Represent in a Number line to show the equal groups that have been subtracted. The arrows go from the dividend to zero. The number of jumps equals the number of groups.	Children are introduced to the ÷ sign. 12 ÷ 4 = 3 12 ÷ 3 = 4 This is linked to the Number line. 12 – 4 – 4 – 4 = 0 12 – 3 – 3 – 3 – 3 = 0 Discuss how division is not commutative e.g. 12 ÷ 3 = 4 but 3 ÷ 12 doesn't = 4 However, 12 ÷ 3 = 4 and 12 ÷ 4 = 3!

	Use a bead string to help children to group. 12 ÷ 3 = 4 		
Linking multiplication and division through missing number questions	Use objects to make 24. I know there are 2 lots so split them up. How many in each group? 2 x ? = 24 	Drawing arrays or groups: 3 X ? = 12 	Introducing the Inverse operations Trios can be used to model the 4 related multiplication and division facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.
Children should be here by the end of Y2			

Times tables Year 2

Objectives	**Concrete**	**Pictorial**	**Abstract**
Recall 2s 5s and 10s time's tables.	Using a counting stick to count forwards and backwards in 2s, 5s and 10s. Explore the patterns they notice by taking off the post it note each time to discover the next number.  	Children are given a number line with numbers missing or numbers in the wrong order and they are asked to fill them in or circle which is in the wrong order. For example;  Or  	Children are asked to see the patterns in the 2s, 5s and 10s time's tables. For example, in the 2 times table all of the numbers are even. For example;